CHAPTER

1

Bird's-Eye View Approach to Algebra

1.1 :: Factor Out!

try it yourself Try these four sample questions within 60 seconds.

Q1 If $50^{100} = 2^{100} \bullet 5^{200}x$, what is x?

Q2 If $a - b = 5$ and $2ab - 2b^2 = 100$, what is b?

Q3 If $x^2 - y^2 = mn$, and $x + y = n$, what is $x - y$?

Q4 If $\frac{9^8 - 9^7}{8} = x^7$, then x?

general rules

☑ First, look for **what the question wants**.
☑ Before using the conventional method of variable elimination, try the **"Bird's-Eye View" method of factoring**, utilizing the four essential factoring formulas:

Number factoring: $12 = 2 \bullet 2 \bullet 3 = 2^2 \bullet 3$

Monomial factoring: $2x^2y^2 - 4xy^3 + 2xy = 2xy(xy - 2y^2 + 1)$

Binomial factoring: $4a^2 - 9b^2 = (2a + 3b)(2a - 3b)$

Trinomial factoring: $2x^2 + 3x + 1 = (2x + 1)(x + 1)$

approach to sample questions

A1 Factoring 50^{100} gives us $(2 \bullet 5^2)^{100} = 2^{100} \bullet 5^{200}$. Therefore, x = 1.

A2 A simple monomial factoring of $2ab - 2b^2$ yields $2b(a - b) = 100$.
Since a - b = 5, dividing both sides by 5 yields 2b = 20. Therefore, b = 10.

A3 Through binomial factoring, $x^2 - y^2 = (x + y)(x - y)$.
Since x + y = n, x - y has to be m.

A4 The simple factoring of $9^8 - 9^7$ yields $9^7(9 - 1) = 9^7(8)$.
Therefore, the answer for x is 9.

Practice Questions 1.1

1 If $7(x + y) - 4(x + y) = 15$, then x + y = ?

2 If x + y = 7 and x - y = 4, then $x^2 - y^2$ = ?

3 If $x - y = 6$ and $x^2 - xy = 24$, what is y?

4 Simplify $\sqrt{80(11)^2 + (11)^2}$

5 Simplify $\dfrac{7^{17} - 7^{16}}{6}$

6 $\dfrac{10! - 9!}{9!} =$

(A) 10/9
(B) 9
(C) 10
(D) 9!
(E) 10!

3.9 :: Quadratic / Higher-Order Functions, Graphs, & Eqns

try it yourself Try this sample question within 30 seconds.

Q1 Solve $(x + 2)^2 = (x - 3)^2$

general rules

☑ To **graph a quadratic function, transform it:**

If $a > 0$, the parabola opens upwards

If $a < 0$, the parabola opens downwards

General form: $y = ax^2 + bx + c$

to find y-intercept, c

Factored form: $y = a(x - m)(x - n)$

to find x-intercepts, m and n

Complete square form: $y = a(x - h)^2 + k$

to find the vertex (h, k) and the axis of symmetry, $x = h$

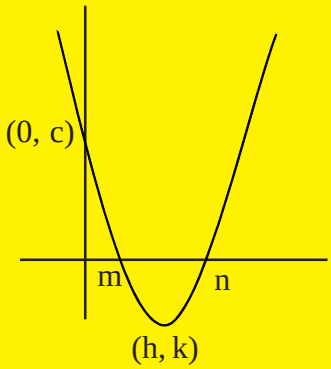
☑ To **solve a quadratic equation**, arrange the equation to the factored form (See Section 3.1) and set it equal to zero on the right-hand side:

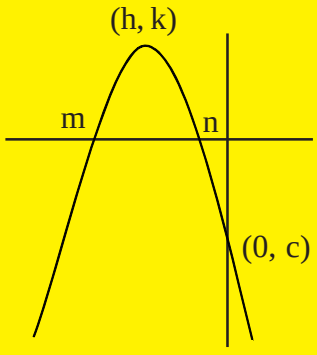
$(x - m)(x - n) = 0$. Then, $x - m = 0$ and $x - n = 0$.

Therefore, $x = m$ or $x = n$.

Short cut: If $(x - m)^2 = 0$, then $x = m$.

If $(x - m)^2 = (x - n)^2$, then $x - m = \pm(x - n)$.





☑ To **find the intersections** of a quadratic function and others, equate the two functions as substitutes for y.

That is, if $y = ax + b$ and $y = cx + d$, equate them: $ax + b = cx + d$.

Most of the quadratic function-related questions in the SAT do not require you to remember the vertex formula. However, you should remember how to read information from the graph of a parabola, such as x-intercepts (when y = 0), the y-intercept (when x = 0), the axis of symmetry (when x = h), and the vertex point among other coordinates.

approach to sample questions

A1 If $(x - h)^2 = (x - k)^2$, then $x - h = \pm(x - k)$. Testing both of these:

$(x + 2) = +(x - 3)$, which can never be true, and

$(x + 2) = -(x - 3)$

So, $2x = 1$ and therefore $x = 1/2$.

Practice Questions 3.9

1 What is/are the value(s) of x satisfying the equation $(x - 1)^2 = (x + 3)^2$?

2 If $x + 3y = 4$ and $x = y^2 + 4$, which of the following is a possible value for y?

(A) -4
(B) -3
(C) 1/3
(D) 1/2
(E) 5

3 If $x + 3$ is a factor of $x^2 - tx - 12$, then $t =$

(A) -4
(B) -1
(C) 1
(D) 3
(E) 5

4 If $x^2 + 5x - 6 = 0$ and $x < 0$, which of the following is not equal to 0?

(A) $x^2 + 4x - 12$
(B) $x^2 - 5x - 6$
(C) $x^2 + 7x + 6$
(D) $x^2 + 4x + 12$
(E) All are equal to 0

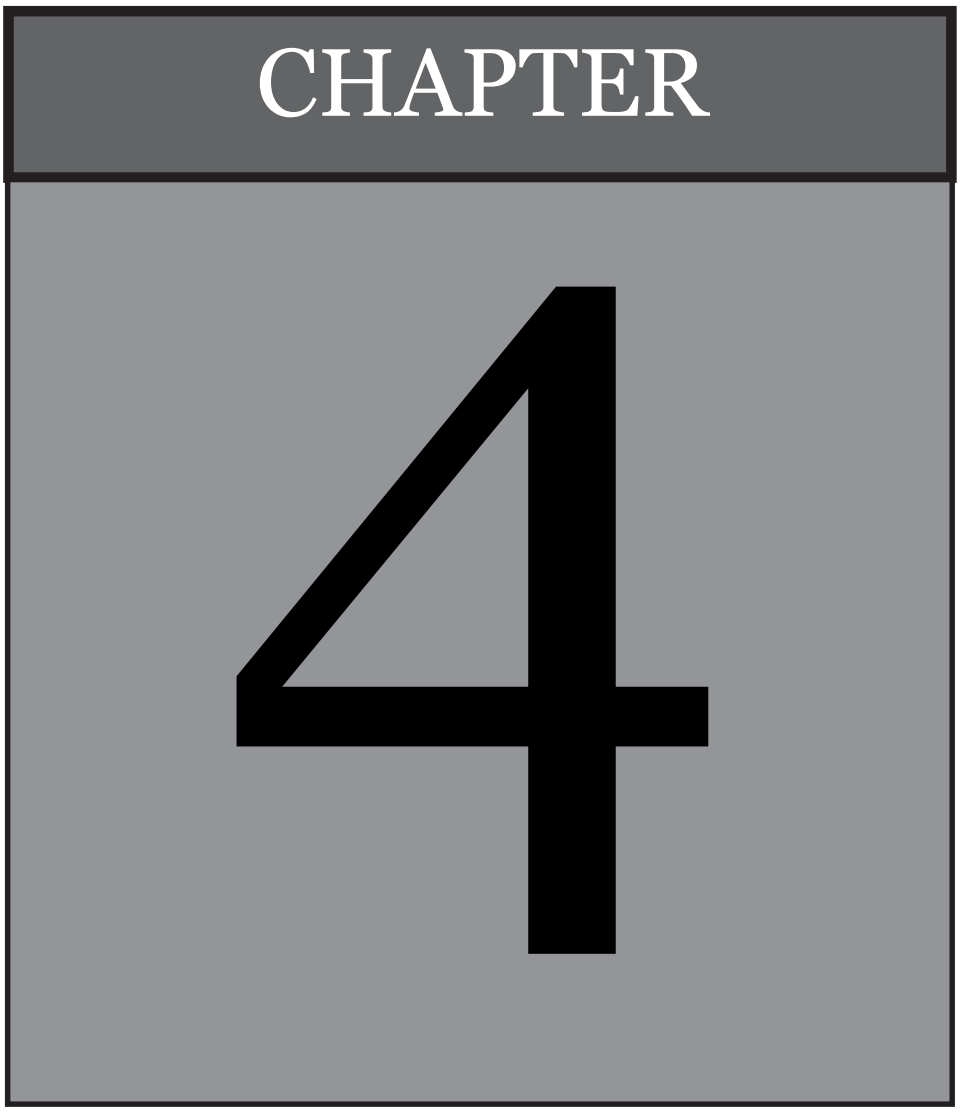
5 If $(x - k)^2 = 9$, what is the sum of the values of the solutions for x?

(A) $k + 9$
(B) $k - 9$
(C) $k + 3$
(D) $k - 3$
(E) $2k$

6 The table below provides values of quadratic function f for some values x. Which of the following equations could define f(x)?

x	-2	-1	0	1	2
f(x)	-3	0	1	0	-3

(A) $f(x) = x^2 + x - 1$
(B) $f(x) = x^2 - 1$
(C) $f(x) = -x^2 + x + 1$
(D) $f(x) = -x^2 + 1$
(E) $f(x) = x^2 - x + 1$



Special Types of Algebra Problems

4.1 :: Extra-Terrestrial Problem

try it yourself Try these two sample questions within 30 seconds.

Q1 If $I \heartsuit U = \frac{I + U}{I - U}$, then what is $5 \heartsuit 10$?

(A) -5
(B) -3
(C) 1
(D) 3
(E) 15

Q2 Let $<n = n/2$ if n is even and $<n = 2n$ if n is odd.
What is $<7 \bullet <20$?

(A) <140
(B) <70
(C) <270
(D) <1400
(E) <280

general rules

Don't panic if you see some strange symbols on the test. This kind of math may seem extra-terrestrial, but with our super intelligent brains, we humans can decipher what these symbols mean.

Example: If $I \heartsuit U = IU + I + U$,
then $3 \heartsuit 4 = 12 + 3 + 4 = 19$.

Example: If @[x] is defined as the least integer greater than or equal to x,
then @[-3.2] = -3

Example: If [x] is 2 more than the number of digits in the integer x,
then [1000] = 4 + 2 = 6.

approach to sample questions

A1 Following the definition, $5 \heartsuit 10 = \frac{5 + 10}{5 - 10} = \frac{15}{-5} = -3$. The answer is (B).

A2 Since <7 is 14 and <20 equals 10, <7 • <20 = 14 • 10 = 140.
Choose the answer that yields 140.
Since <240 = 140, the answer is (E).

Practice Questions 4.1

1 If the operation @ is defined for all numbers x and y by the equation $x @ y = \frac{x - y}{3}$, then 3 @ (11 @ 2) =

(A) -5
(B) -3
(C) 0
(D) 1
(E) 3

2 For any number x, x^ is defined as the greatest integer that is less than x. What is (3.7)^ + (-3)^ ?

(A) -3
(B) -1
(C) 0
(D) 1
(E) 3

3 Let x*** = $x - x^2$ for all non-negative integers x. Which of the following equals $x^2 - x + x***$?

(A) -3***
(B) -1***
(C) 1***
(D) 2***
(E) 3***

CHAPTER

Special Types of Word Problems

6.1 :: Set (Venn Diagram)

try it yourself

Try these two sample questions within 60 seconds.

Q1 Among Ali's class of 40 students, 13 people speak Spanish, 30 people speak English, and 6 people speak both English and Spanish.
1) How many people speak neither language?
2) How many people speak only one language?

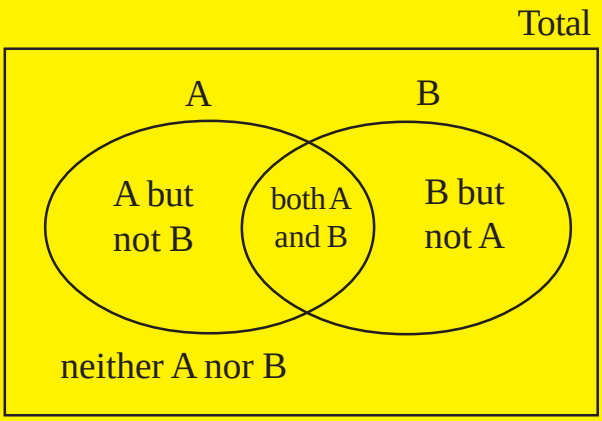
Q2 Of 300 students in Robert's school, 2/3 have PC's and 60 have Macs. Of the students who have PC's, 30 have Macs as well. How many students have neither PC's nor Macs?

general rules

☑ **Venn diagram** approach:
If a person or thing belongs to group A or group B,

Total = m(A or B) + m(neither A nor B)
m(A or B) = m(A) + m(B) - m(A and B)
m(A) + m(B) = m(A or B) + m(A and B)

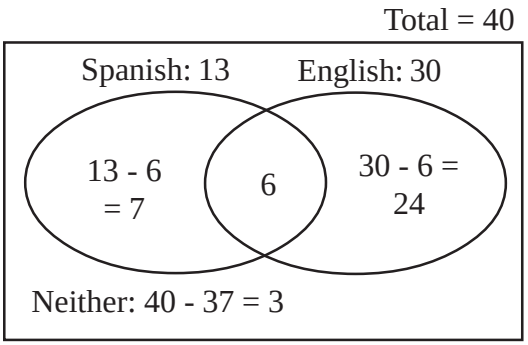
where m represents members.



approach to sample questions

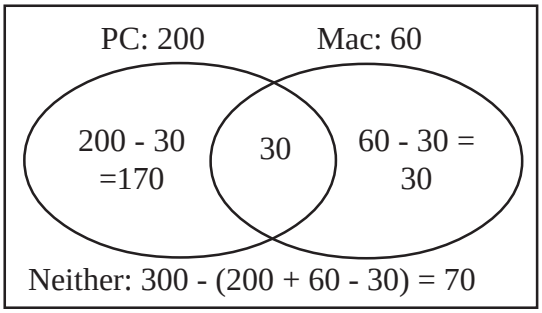
A1

1) m(A or B) = m(A) + m(B) - m(A and B)
= 13 + 30 - 6 = 37
m (neither A nor B) = total - m(A or B)
= 40 - 37 = 3
2) Only one language = 7 + 24 = 31



A2

Total = 300
m(A) = PC owners = 2/3 of 300 = 200
m(B) = Mac owners = 60
m(A and B) = owner of both = 30
m(A or B) = computer owners
= m(A) + m(B) - m(A and B)
= 200 + 60 - 30 = 230
Students without computers = 300 - 230 = 70



Practice Questions 6.1

1 Of 300 pairs of shorts in a certain clothing store, exactly 160 are casual wear and 200 are swimwear. If 80 percent of the casual wear can be used as swimwear, how many of the pairs of shorts are neither casual wear nor swimwear?

2 At Fantasyland High School, there are 16 members in the math club and 12 members in the chess club. If 10 students belong to only one of the two clubs, how many students belong to both clubs?